Assignment 2 for MATH4220

February 4, 2016

Exercise 2.1, 1, 2, 5, 6, 7, 8, 10

Exercise 2.2, 1, 2, 3, 5

Exercise 2.3, 2, 3, 4, 5, 6, 7

More on 2.3:

Extra 1. Consider the diffusion equation $u_t = ku_{xx} + au$ in $(0 < x < 1, 0 < t < \infty)$ with $u(0,t) = u(1,t) = 0$ and $u(x,0) = sin(\pi x)$, where $k > 0, a$ are real numbers.

(1) Show that $0 < u(x,t) < 1$ for all $t > 0$ and $0 < x < 1$.

(2) Show that $u(x,t) = u(1-x,t)$ for all $t \geq 0$ and $0 \leq x \leq 1$.

Extra 2.(a) Prove the following generalized Maximum Principle:

if $u_t - ku_{xx} \leq 0$ in $R = [0,l] \times [0,T]$, then

$\max\limits_{R} u(x,t) = \max\limits_{\partial R} u(x,t)$

Hint: follow the proof of Maximum Principle.

(b) Show that if $v(x,t)$ satisfies

$v_t = kv_{xx} + f(x,t), -\infty < x < +\infty$, $0 < t < T$

$v(x,0) = 0$

then $v(x,y) = \leq T \max\limits_{-\infty < x < +\infty, 0 < t < T} f(x,t)$.

Hint: consider

$u(x,t) = v(x,t) - t \max\limits_{-\infty < x < +\infty, 0 < t < T} f(x,t)$

and then use (a).

Exercise 2.4, 1, 2, 5, 8, 11, 14, 15, 16, 18

Exercise 2.5, 1